MTISD 2008 – Methods, Models and Information Technologies for Decision Support Systems
Università del Salento, Lecce, 18-20 September 2008

Longevity Risk and Reinsurance Strategies for Enhanced Pensions

Susanna Levantesi
University of Rome "La Sapienza"
susanna.levantesi@uniroma1.it

Massimiliano Menzietti
University of Calabria
massimiliano.menzietti@unical.it

Abstract: *This paper deals with demographic risk analysis in Enhanced Pensions, i.e. Long Term Care (LTC) insurance covers for retired. Both disability and longevity risk affect such a cover. Specifically, we concentrate on the risk of systematic deviations between projected and realised mortality and disability, adopting a multiple scenarios approach. To this purpose we study the behaviour of the random risk reserve. Moreover, we analyse the effect of demographic risk on Risk-Based Capital requirements considering different time horizons and confidence levels and explaining how they can be reduced through either safety loading and reinsurance strategy. A profit analysis is also considered.*

Keywords: Demographic risks, Long Term Care covers, risk reserve, risk-based solvency requirements, profit analysis, reinsurance treaty

1. Actuarial model

The "Enhanced Pension" (EP) is a Long Term Care (LTC) insurance cover for retired. It offers an immediate life annuity increased once the insured becomes LTC disabled and requires a single premium. EP is affected by demographic risks (longevity and disability risk) arising from the uncertainty in future mortality and disability trend that causes the risk of systematic deviations from the expected values. To evaluate such a risk we carry out an analysis taking into account a multiple scenario approach.

The probabilistic framework of an EP is defined consistently with a continuous and inhomogeneous multiple state model (see Haberman-Pitacco (1999)). Let $S(t)$ represent the random state occupied by the insured at time t, for any $t \geq 0$, where t is the policy duration and 0 the time of entry. The possible realizations of $S(t)$ are: 1 = "active" (or healthy), 2 = "LTC disabled", 3 = "dead". We disregard the possibility of recovery from the LTC state due to the usually chronic character of disability and we assume $S(0)=1$. Let us define transition probabilities and intensities:

$$P_{ij}(t,u) = \Pr\{S(u)=j \mid S(t)=i\} \qquad 0 \leq t \leq u, \quad i,j \in \{1,2,3\} \tag{1}$$

$$\mu_{ij}(t) = \lim_{u \to t} \frac{P_{ij}(t,u)}{u-t} \qquad t \geq 0, \quad i,j \in \{1,2,3\},\ i \neq j \tag{2}$$

EPs are single premium covers providing an annuity paid at an annual rate $b_1(t)$, when the insured is healthy and an enhanced annuity paid at an annual rate $b_2(t) > b_1(t)$, when the insured is LTC disabled. Let us suppose all benefits to be constant with time. Let ω be the maximum policy duration related to residual life expectancy at age x and let $v(s,t) = \prod_{h=s+1}^{t} v(h-1,h)$ be the value at time s of a monetary unit at time t; the actuarial value at time 0 of these benefits, $\Pi(0,\omega)$, is given by:

$$\Pi(0,\omega) = b_1 a_{11}(0,\omega) + b_2 a_{12}(0,\omega) \quad (3)$$

where: $a_{ij}(t,u) = \sum_{s=t}^{u-t-1} P_{ij}(t,s) v(s,t)$ for all i, j $\in 1,2$. Assuming the equivalence principle, the gross single premium paid in $t=0$, Π^T, is defined as:

$$\Pi^T = \frac{\Pi(0,\omega)}{1-\alpha-\beta-\gamma[a_{11}(0,\omega)+a_{12}(0,\omega)]} \quad (4)$$

where α, β and γ represent the premium loadings for acquisition, premium earned and general expenses, respectively.

2. Demographic scenarios

A risk source in actuarial evaluations is the uncertainty in future mortality and disability; to represent such an uncertainty we adopt different projected scenarios. We start from a basic scenario, H_B, defined according to the most recent statistical data about people reporting disability (see ISTAT (2005)) and, consistently to such data, the Italian Life-Table SIM-2004. Actives' mortality $\mu_{13}(t)$ is approximated by the Weibull law, while transition intensities $\mu_{12}(t)$ by the Gompertz law (for details about transition intensities estimation see Levantesi-Menzietti (2007)). Disabled mortality intensity $\mu_{23}(t)$ is expressed in terms of $\mu_{13}(t)$ according to the time-dependent coefficient $K(t)$, $\mu_{23}(t) = K(t)\mu_{13}(t)$. Values of $K(t)$, coming from the experience data of an important reinsurance company, are well approximated by the function $\exp(c_0 + c_1 t + c_2 t^2)$.
Mortality of projected scenarios has been modelled evaluating a different set of Weibull parameters (α, β) for each ISTAT projection (low, main and high hypothesis, see ISTAT (2002)). Furthermore, the coefficient $K(t)$ is supposed to be the same for all scenarios. As ISTAT data show a decrement in transition intensity, $\mu_{12}(t)$, between 2000 and 2005, (see ISTAT (2000) and ISTAT (2005)), we define three different sets of Gompertz parameters starting from basic scenario to represent three different scenarios for transition intensities: a weak (hp.a), a medium (hp.b) and a strong reduction (hp.c). By combining mortality and disability projections we obtain nine scenarios.

3. A risk theory model

Demographic risk analysis is carried out on a portfolio of EPs with $N_i(t)$ contracts in state i at time t, closed to new entries. The random risk reserve is adopted as risk measure. Let $U(0)$ be the value of the risk reserve at time 0, the risk reserve at the end of year t is defined as:

$$U(t) = U(t-1) + P^T(t) + J(t) - E(t) - B(t) - \Delta V(t) + K(t) \quad (5)$$

where $P^T(t)$ is the gross single premiums income; $J(t)$ are the investment returns on assets, $A(t)$, where the assets are defined as $A(t) = A(t-1) + P^T(t) - E(t) - B(t) + J(t)$; $E(t)$ are the expenses; $B(t)$ is the outcome for benefits; $\Delta V(t)$ is the annual increment in technical provision; $K(t)$ are the capital flows; if $K(t) > 0$ the insurance company distributes dividends, if $K(t) < 0$ stockholders invest capital. We assume that premiums, benefits, expenses and capital flows are paid at the beginning of each year.
The risk analysis is performed according to a multiple scenarios approach that considers each scenario as a possible state according to a probability vector, allowing to evaluate the risk of

systematic deviations in biometric functions (see Olivieri-Pitacco (2001) and Levantesi-Menzietti (2007)). Demographic pricing basis is defined according to central scenario with a safety loading given by a reduction of death probabilities. We disregard financial risk, adopting a deterministic and constant interest rate. We assume financial pricing basis equal to the real-world one.

3.1 Risk-based capital requirements

Risk-Based Capital (RBC) is a method for assessing the solvency of an insurance company; it consists in computing capital requirements that reflect the size of overall risk exposures of an insurer. Let us consider RBC requirements based on risk reserve distribution. We calculate RBC requirements with different time horizons and confidence levels. Let us define the finite time ruin probability as the probability to be in ruin state at least in one of the time points 1, 2..., T, for a given $U(0)=u$:

$$\Psi_u(0,T)=1-\Pr\left\{\bigcap_{t=1}^{T}U(t)\geq 0\,\middle|\,U(0)=u\right\} \tag{6}$$

RBC requirements for the time horizon $(0,T)$ with a $(1-\varepsilon)$ confidence level are defined as follows:

$$RBC_{1-\varepsilon}(0,T)=\inf\{U(0)\geq 0|\Psi_0(0,T)<\varepsilon\} \tag{7}$$

Note that the risk reserve must be not negative for all $t\in(0,T)$.

An alternative method to calculate RBC requirements is based on the Value-at-Risk (VaR) of theU-distribution in the time horizon $(0,T)$ with a $(1-\varepsilon)$ confidence level: $VaR_{1-\varepsilon}(0,T)=-U_\varepsilon(T)$ where $U_\varepsilon(T)$ is the ε-th quantile of the U-distribution at time t. Hence RBC requirements are given by:

$$RBC_{1-\varepsilon}^{VaR}(0,T)=VaR_{1-\varepsilon}(0,T)v(0,T) \tag{8}$$

If an initial capital $U(0)$ is given, the $RBC_{1-\varepsilon}^{VaR}(0,T)$ requirements increase of the amount $U(0)$.

3.2 Profit analysis

We also analyse the annual profit, $Y(t)$, emerging from the management of the portfolio. In order to catch the profit sources, $Y(t)$ can be decomposed in insurance profit:

$$Y^I(t)=(1+i(t-1,t))[V(t-1)+P^T(t)-E(t)-B(t)]-V(t) \tag{9}$$

and in profit coming from investment income on shareholders' funds ("patrimonial profit"), $Y^P(t)=U(t-1)i(t-1,t)$. The sequence $\{Y(t)\}_{t\geq 1}$ is called profit profile.

4. Reinsurance treaty

Many reinsurance strategies can be considered to carry mortality and disability risk on a tolerable level. In insurance practice LTC risk is usually covered by quota share treaties. Nonetheless, it is well-known that among reinsurance treaties stop-loss (SL) gives the smallest variance of the insurer's retained risk. The cedant takes on the risk up to a certain amount, after which reinsurance begins. Such kind of risk is tail-end and represents the most volatile part of a EP.

In this paper the SL reinsurance is developed assuming that the reinsurer's intervention is linked to the loss sustained by the insurer equal to the excess of annuities payment respect to expected values at portfolio level. We consider a SL treaty covering k years, so for longer periods reinsurance arrangements are set every k years. Let r be a coefficient defining the reinsurance excess limit (retention) and $E_h[B(t)]$ the annuities the insurer expects to pay in period $t \in \{h+1, h+2, ..., h+k\}$ considering information available at time h. The possible payment from the reinsurer at the end of the year t, $B^{SL}(t)$, is defined as $\max\{0; B(t)-(1+r)E_h[B(t)]\}$.:

When the SL treaty is in force the risk reserve at time $h+k$ becomes:

$$U^{SL}(h+k) = U(h+k) + \sum_{s=1}^{k}\left(\frac{B^{SL}(h+s)}{v(h+s,h+k)}\right) - \frac{P^{SL}(h)}{v(h,h+k)} \qquad (11$$

where $P^{SL}(h)$ represent the reinsurance premium.

5. Simulation results and conclusions

Simulated values of the risk reserve distribution shows a strong variability. Even though the risk reserve has a positive trend due to safety loading, lower percentiles are negative. Economic consequences of such an aspect are relevant for the insurer solvency and will be quantified through solvency requirements. We also analyse the expected values of annual profit components: the insurance profit shows greater variability being affected by demographic risks, while the patrimonial profit line is more regular due to the absence of financial risk, and increases with time, depending by investments on risk reserve. When a highest safety loading is considered, the probability of risk reserve to become negative decreases, but its variability not lowers. Moreover, required capital decrease at safety loading increase. Positive effects of reinsurance on risk are recognizable when we introduce a SL reinsurance treaty.

In conclusion our analysis highlights that the EPs are affected by a significant demographic risk caused by systematic deviations between expected and realized demographic scenarios. Results confirm that such a risk is difficult to control: the u(t) variability does not lessen when either safety loading or initial capital increase. Reinsurance seems more effective on reducing RBC requirements variability. In this paper we take into account an initial capital only, reducing the ruin probability of the insurance company as far as the RBC requirements. Anyhow, the risk of systematic deviations persists, requiring an appropriate capital allocation strategy. This topic will be object of future researches considering the effects of each capital allocation strategy on profitability.

Bibliography

Haberman S., Pitacco E. (1999) Actuarial models for Disability Insurance, Chapman-Hall, London

ISTAT (2000), Le condizioni di salute e ricorso ai servizi sanitari, Settore Famiglie e società, Istituto Nazionale di Statistica, Roma.

ISTAT (2002), Previsioni della popolazione residente per sesso età e regione dal 1.1.2001 al 1.1.2051, Settore Popolazione, Istituto Nazionale di Statistica, Roma

ISTAT (2005), Le condizioni di salute e ricorso ai servizi sanitari, Settore Famiglie e società, Istituto Nazionale di Statistica, Roma

Levantesi S., Menzietti M. (2007), Longevity and disability risk analysis in enhanced life annuities, Proceeding of the 1st International LIFE Colloquium, Stockholm

Olivieri A., Pitacco E. (2001), Facing LTC risks, Proceedings of the 32nd International ASTIN Colloquium, Washington

Olivieri A. (2002), Risk and reinsurance in life annuity portfolios, Working paper n.3, Dipartimento di Matematica Applicata alle Scienze Economiche, Statistiche ed Attuariali, University of Trieste

Rytgaard M., Savelli N. (2004), Risk-Based Capital Requirements for Property and Liability Insurers according to different Reinsurance Strategies and the Effect on Profitability, Proceedings of the 35th International ASTIN Colloquium, Bergen